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LETTER TO THE EDITOR

The effect of dislocated cracks on collective excitations in a superlattice

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Abstract. The detection of dislocated cracks hidden in a superlattice is a very difficult problem of considerable practical importance. The calculation performed here proves that splitting of coupled localised edge modes occurs due to the presence of the dislocated cracks. The coupled anomalous edge modes may be 'softened' in a system which facilitates complete Coulomb screening—a superlattice with dislocated cracks, bulk with cracks, etc. This provides an approach for detecting dislocated cracks hidden in a superlattice.

Recently, results were reported on intra-sub-band surface plasmon modes on the lateral surface of a half-plane semiconductor superlattice, which were called 'edge modes' in such a system [1]. The edge magnetoplasmon modes on a lateral surface of a half-plane superlattice with a complex unit cell and the coupled edge magnetoplasmon modes in a plane with a ditch have also been studied [2, 3]. It has further been pointed out that 'softening' of the coupled edge plasmons occurs on lateral surfaces of the superlattice with cracks as well as in the bulk with cracks [4]. The existence of a magnetic field suppresses the softening of these anomalous edge modes. The detection of the cracks hidden in the superlattice is a problem of practical importance for surface-wave devices. Usually, these cracks are accompanied by dislocations in the superlattice.

Here we use a hydrodynamic model to study the magnetoplasmon modes in a superlattice with dislocated cracks. For generalisation, we use a model in which periodic arrays of dislocated 2D electron layers are stacked along the z direction, and the dislocated electron layers are respectively located in the spaces $x < 0$ (region 1) and $x > a$ (region 2) separated by a distance a and embedded in a semiconductor background of dielectric constant ε_s. The superlattice spacing is d. The external magnetic field is along the z direction perpendicular to the half-planes.

Our main interest is the self-consistent oscillation of a charge-compensated 2D electron confined between the dislocated layers respectively situated at $x < 0$, and $x > a$, placed in a perpendicular magnetic field Bz^0. Consider a rigid positive background with charge density en_0 and a compressible electron fluid with number density $n_0 + n$. Let $n_j(r, t)$ and $v_j(r, t)$ denote, respectively, the small fluctuation in the electron surface density and the electron velocity field in the plane of the jth layer. These amplitudes satisfy the equation of continuity, Euler's equations and Poisson's equation:

$$-\mathrm{i}\omega n_j + n_0(\partial v_{jx}/\partial x + \mathrm{i}kv_{jy}) = 0 \qquad (1)$$

$$-\mathrm{i}\omega v_{jx} + (s^2/n_0)\partial n_j/\partial x - (e/m^*)\partial\varphi/\partial x + \omega_c v_{jy} = 0 \qquad (2)$$

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$$-\mathrm{i}\omega v_{jy} + \mathrm{i}ks^2(n_j/n_0) - \mathrm{i}k(e/m^*)\varphi - \omega_c v_{jx} = 0 \tag{3}$$

$$(\partial^2/\partial x^2 + \partial^2/\partial z^2 - k^2)\varphi(x,z) = (4\pi e/\varepsilon_s)\sum_j [n_j(x)\delta(z-z_j)\theta(-x) + n_j'(x)\delta(z-z_j-\nu d)\theta(x-a)] \tag{4}$$

where φ is the electrostatic potential, ω_c is the cyclotron frequency, and $0 \leqslant \nu \leqslant 1$ is the dislocation parameter. θ is the step function, and s is an effective compressional wave speed. Since the system is translationally invariant along the y direction, the solution may be taken as a plane wave or the form $\exp(\mathrm{i}ky - \mathrm{i}\omega t)$, with amplitudes dependent on x and z. A Fourier transform in x of equation (4) leads to the ordinary differential equation

$$[\mathrm{d}^2/\mathrm{d}z^2 - (k^2 + k_x^2)]\varphi(k_x, z) = (4\pi e/\varepsilon_s)\sum_j [n_j(k_x)\delta(z-z_j) + n_j'(k_x)\delta(z-z_j-\nu d)] \tag{5}$$

where $n_j(k_x)$ and $n_j'(k_x)$ are the Fourier transforms of $n_j(x)\theta(-x)$ and $n_j(x)'\theta(x-a)$, respectively. Its solution can be expressed as

$$\varphi(k_x, z) + (2\pi e/\varepsilon_s)\sum_j (k')^{-1}[n_j(k_x)\exp(-k'|z-z_j|) + n_j'(k_x)\exp(-k'|z-z_j-\nu d|)] = 0 \tag{6}$$

where $k' = (k^2 + k_x^2)^{1/2}$. The inverse Fourier transform then produces a non-local integral relation between the electrostatic potential in the lth plane and the corresponding charge density

$$\varphi(x, z_l) + (4\pi e/\varepsilon_s)\sum_j \int \mathrm{d}x'\, L_j(x-x')[n_j(x')\theta(-x') + n_j'(x')\theta(x'-a)] = 0 \tag{7}$$

where

$$L_j(x) = \int \mathrm{d}k_x \exp(\mathrm{i}k_x x)(2k')^{-1}[\exp(-k'|z_l - z_j|) + \exp(-k'|z_l - z_j - \nu d|)]. \tag{8}$$

In principle, such an integral equation can be solved by using the Wiener–Hopf technique [5]. In analogy with the density fluctuations for a classical one-dimensional harmonic lattice, the density fluctuations on the layers at $z_j = jd$ and $z_j = jd + \nu d$ can be related to those on the zeroth layer with the help of the usual Bloch condition

$$n_j(x') = A_1(x')\exp(\mathrm{i}q_z jd) \tag{9a}$$

$$n_j'(x') = A_2(x')\exp[\mathrm{i}q_z(j+\nu)d] \tag{9b}$$

where q_z is the wavevector perpendicular to the plane, and the amplitudes $A_1(x')$ and $A_2(x')$ are independent of the layer levels j. Then equations (7) and (8) give

$$\varphi(x, z_1) + (2\pi e/\varepsilon_s)\int \mathrm{d}k_x \exp(\mathrm{i}k_x x)(k')^{-1}[A_1(k_x)S_1(k_x,k,q_z) + A_2(k_x)S_2(k_x,k,q_z)] = 0 \tag{10a}$$

with

$$S_1(k_x,k,q_z) = \sum_j \exp[-k'|z_l - jd| + \mathrm{i}q_z(jd)] \tag{10b}$$

$$S_2(k_x,k,q_z) = \sum_j \exp[-k'|z_l - jd - \nu d| + \mathrm{i}q_z(jd+\nu d)] \tag{10c}$$

$$A_{1,2}(k_x) = \int \mathrm{d}x' \exp(-\mathrm{i}k_x x')A_{1,2}(x'). \tag{10d}$$

These are independent of the layer levels. As in [13], after a lengthy manipulation, we obtain the Fourier component of the exact kernel about equation (7) using the combination of equations (10*a*)–(10*d*)

$$L(k_x) = [2(k_x^2 + k^2)^{1/2}]^{-1} S(k_x, k, q_z) \tag{11a}$$

where the screening function is

$$\begin{aligned} S(k_x, k, q_z) = [\![&\sinh(k'd) \pm \{\sinh[k'(1-\nu)d] + \sinh(k'\nu d) \\ &+ 2\cos(q_z d)\sinh[k'(1-\nu)d] \\ &\times \sinh(k'\nu d)^2\}]\!]/[\cosh(k'd) - \cos(q_z d)]. \end{aligned} \tag{11b}$$

We introduce the approximation used in [6], which we think remains applicable and will be seen to work with equal ease [2, 3]; then we obtain

$$L_0(k_x) = kf(k, q_z)/[2k^2 + k_x^2 g(k, q_z)] \tag{12}$$

where

$$g(k, q_z) = 1 - [k/f(k, q_z)][\mathrm{d}f(k, q_z)/\mathrm{d}k]. \tag{13}$$

The function $g(k, q_z)$ characterises the screening correction for edge plasmons, and $f(k, q_z) = S(k_x = 0, k, q_z)$. $L(k_x)$ and $L_0(k_x)$ have the same first two terms in a power series about $k_x^2 = 0$. The inverse Fourier transform of equation (12) yields the approximate kernel

$$L_0(x) = 2^{-1} f(k, q_z)(2g)^{-1/2} \exp[-(2/g)^{1/2} k|x|]. \tag{14}$$

This problem can be reduced to three effective localised Poisson equations

$$(\mathrm{d}^2/\mathrm{d}x^2 - 2k^2/g)\varphi_1(x, z_l) = (4\pi ek/\varepsilon_s) fg^{-1} \sum_j n_j(x) \qquad (x < 0) \tag{15}$$

$$(\mathrm{d}^2/\mathrm{d}x^2 - 2k^2/g)\varphi_2(x, z_l) = (4\pi ek/\varepsilon_s) fg^{-1} \sum_j n_j'(x) \qquad (x > a) \tag{16}$$

$$(\mathrm{d}^2/\mathrm{d}x^2 - 2k^2/g)\varphi_3(x, z_l) = 0 \qquad (0 < x < a). \tag{17}$$

The remaining steps in the solution are identical with those in [2] and [4]. After combining equations (1)–(3) and (15)–(17) with the boundary conditions that φ and $\partial\varphi/\partial x$ are continuous and that v_x vanishes at the boundary, together with the suitable boundary behaviour $|x| \to \infty$, this procedure gives the dispersion relation:

$$\begin{aligned} D^4\omega^2\{&2(2/g)^{1/2} C \sinh[(2/g)^{1/2}ka] + C^2 \sinh[(2/g)^{1/2}ka] + (2/g)\sinh[(2/g)^{1/2}ka]\} \\ &- 4(2/g)^{1/2} D^2 \omega_k^2 \omega^2 (f/g)\{C\cosh[(2/g)^{1/2}ka] \\ &+ (2/g)^{1/2}\cosh[(2/g)^{1/2}ka]\} + 4\omega_k^4 (f/g)^2 \\ &\times \{(2/g)\omega^2 \sinh[(2/g)^{1/2}ka] - \omega_c^2 \sinh[(2/g)^{1/2}ka]\} = 0 \end{aligned} \tag{18}$$

where $\omega_k^2 = 2\pi n_0 e^2 k/m^*$ is the bulk 2D plasma frequency, and

$$D^2 = 2\omega_k^2(f/g) + (\omega_c^2 - \omega^2) \tag{19}$$

$$C^2 = 2[(\omega_k^2(f/g) + (1/g)(\omega_c^2 - \omega^2)]/[2\omega_k^2(f/g) + (\omega_c^2 - \omega^2)]. \tag{20}$$

An additional set of roots are given by $\omega^2 = \omega_c^2$ (spurious result of the approximation method) and $\omega^2 = [2\omega_k^2(f/g) + \omega_c^2]$ (corresponding to the bulk continuum when $a \to 0$) [2].

Equation (17) can readily be solved by a numerical method to give the desired edge plasmon spectrum and the effect of magnetic field on the frequency of edge magnetoplasmons. Although we do not perform such a numerical calculation here, it presents no fundamental difficulty. On the basis of the results shown in figure 1 of [4] (in which there is no dislocation, or $\nu = 0, 1$), we can easily predict the features of the spectrum. The symbol ($\pm$) in equation (11*b*) stands for the interaction between the dislocated charged layers, similar to that in the superlattice partly composed of a complex unit cell.

In general, in the absence of a magnetic field, we have four split branches of coupled edge plasmon modes when $(ka)^{-1} \neq 0$, due to the dislocated cracks hidden in the superlattice ($\nu \neq 0, 1$), to be compared with those given in figure 1 of [4]. When $\nu = 0$ or $\nu = 1$, however, the splitting vanishes. The splitting reaches a maximum when $\nu = \frac{1}{2}$. In addition, the frequency of anomalous edge modes will decrease rapidly when a becomes small; this is called the 'softened' plasmon mode. This can be attributed to the dramatic enhancement of the Coulomb screening due to the strong coupling.

In the presence of a magnetic field, the symmetry with respect to $+y$ and $-y$ directions is broken. When $(ka)^{-1} = 0$, two degenerate branches will be further split. Moreover, the magnetic field will suppress the softening of anomalous edge modes.

Investigation of the splitting of softened edge plasmon modes in a superlattice with dislocated cracks looks attractive, but no reports are available as yet. From the analysis given above, we know that the softening of coupled anomalous edge plasmon modes will occur in a system which facilitates complete Coulomb screening—a superlattice with dislocated cracks, bulk material with cracks, etc. This provides an approach for detecting the dislocated cracks hidden in superlattices. The theory for a model in which only several charged layers are cracked will be given in a separate paper.

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